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MTH 530, Abstract Algebra I (graduate) Fall 2012 ,HW number FOUR (Due: Sat. at 1pm November 3)

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QUESTION 1. (i) Assume that G is a group and $a \in G$ such that $ab = ba$ for some $b \in G$. Prove that $ab^{-1} = b^{-1}a$.

(ii) We need this concept: Let G be a group and define $Z(G) = \{a \in G | ab = ba$ for every $b \in G\}$. Z(G) is called the center of the group G (i.e. Z(G) is the set of all elements in G where each element in Z(G) commutes with every element in G) . Prove that $Z(G)$ is a normal subgroup of G

(iii) Let G be a group such that $G/Z(G)$ is a cyclic group. Prove that G is abelian group.

(iv) Prove that A_4 does not have a normal subgroup of order 3.

(v) We know A_n is simple when $n \geq 5$. Prove that $K = \{(1), (1\ 2)o(3\ 4), (1\ 3)0(2\ 4), (1\ 4)o(2\ 3)\}$ is a normal subgroup of A_4 and hence A_4 is not simple. Construct a nontrivial group homomorphism f from A_4/K into A_3. Find Range(f), Ker(f).

(vi) If D is a normal subgroup of a group G and F is a subgroup of D, then F needs not be a normal subgroup of G. However, prove the following: Let D be a cyclic normal subgroup of a group G and let F be a subgroup of D. Prove that F is a normal subgroup of G.

(vii) (nice problem): Let G be a group of order $2q$ for some prime number $q \geq 3$ and assume that G has a normal subgroup of order 2. Prove that G is a cyclic group.

(viii) Assume that a group G has EXACTLY 10710 elements of of order 11. How many subgroups of order 11 does G have?

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